

2(b) Repeating the analysis with

(4)

$$u(t) = \begin{cases} 0 & t \leq 0 \\ 1 - e^{-5t}, & t > 0 \end{cases}$$

$$\frac{dy}{dt} = 5e^{-5t}$$

$$y(t) = \int_0^t \frac{5}{\beta} \left(1 - e^{-\frac{\beta}{m}(t-\tau)} \right) e^{-5\tau} d\tau$$

$$= \frac{5}{\beta} \int_0^t e^{-5\tau} - e^{-\frac{\beta}{m}t + (\frac{\beta}{m} - 5)\tau} d\tau$$

$$= \frac{5}{\beta} \left[-\frac{1}{5} e^{-5\tau} - \frac{1}{(\frac{\beta}{m} - 5)} e^{-\frac{\beta}{m}t + (\frac{\beta}{m} - 5)\tau} \right]_0^t$$

$$= \frac{5}{\beta} \left[-\frac{1}{5} e^{-5t} + \frac{1}{5} - \frac{1}{(\frac{\beta}{m} - 5)} e^{-5t} + \frac{1}{(\frac{\beta}{m} - 5)} e^{-\frac{\beta}{m}t} \right]$$

$$\frac{1}{\frac{\beta}{m} - 5} = -0.21, \quad \frac{\beta}{m} = 0.24 \frac{1}{5}$$

$$y(t) = 35.3 \left(1 + 0.05 e^{-5t} - 1.05 e^{-0.24t} \right) \text{ m/s}$$

2(c) The responses are both plotted in Figure 3.

Again, the size of the input is such that the linearization assumptions will likely not hold. However the shape of the response makes physical sense: the rocket responds more quickly to the forcing with $t_s = \frac{1}{5}$ (smaller time constant means faster rise).

Also, the response with $t_s = \frac{1}{5}$ is much closer to the step response than the response with $t_s = 1$.